ATMOSPHERIC INVERSIONS: CARBON ISOTOPES

Ian G. Enting
CSIRO Atmospheric Research, PMB 1, Aspendale, Vic 3195, Australia
ian.enting@dar.csiro.au

Issues

In presenting a review of inversions of atmospheric CO_2 at a WOCE/JGOFS meeting it seems appropriate to focus on two questions:
What can the atmospheric CO_2 and its isotopic composition tell us about the ocean carbon cycle?
What can the ocean tell us about the atmospheric carbon budget?

In order to answer these questions we need to be able to quantify the extent to which different types of observations contribute to improved understanding.

The first step is to clarify some definitions [14]. In atmospheric studies we have learnt the importance of

carbon dioxide flux $\neq$ carbon flux $\neq$ carbon storage

The first inequality reflects release from terrestrial biomass to the atmosphere in forms other than CO_2 - generally CO and its precursors. The second reflects the transfer of carbon from terrestrial systems to the oceans via rivers [20]. These differences help explain apparent anomalies such as the very low estimates [12] of oceanic uptake of CO_2.

Atmospheric inversion with isotopes is presented in terms of (a) atmospheric CO_2 inversions, (b) global ^{13}C-^{12}C budgeting and then (c) combining these in joint CO_2-$^{13}CO_2$ inversions.

Inversions

The principle that underlies trace gas inversions is that the spatial and temporal distributions of trace gas concentrations reflect the spatial and temporal distributions of their sources and sinks, modified by the effects of atmospheric transport. Therefore, using a model to calculate the atmospheric transport, it should be possible to identify the sources and sinks. There are a range of techniques [4.5]. The two main types are:
Mass balance inversions. These are based on taking the transport equation expressing changes in concentration, $c(\mathbf{r},t)$, as the combination of transport, $T[.]$ and the net source/sink, $s(\mathbf{r},t)$:

$$\frac{d}{dt}c(\mathbf{r},t) = T[c(\mathbf{r},t)] + s(\mathbf{r},t)$$

and rewriting it as

$$s(\mathbf{r},t) = \frac{d}{dt}c(\mathbf{r},t) - T[c(\mathbf{r},t)]$$

where the second form is used at those locations (generally the earth's surface) where the concentration is known and the source is unknown. The first form is used (generally in the free atmosphere) at those locations where the source is known (often zero) and the concentrations are unknown.

Synthesis inversions. These are based on the integral form

$$c(\mathbf{r},t) = \int G(\mathbf{r},t,\mathbf{r}',t')s(\mathbf{r}',t')d^3r'dt'$$

The sources are discretised in terms of prescribed basis functions, $\sigma_j(\mathbf{r},t)$, as

$$s(\mathbf{r},t) = \sum_j a_j\sigma_j(\mathbf{r},t)$$

and the coefficients, a_j, estimated by

$$c(\mathbf{r},t) \approx \sum_j a_jG_j(\mathbf{r},t)$$

with

$$G_j(\mathbf{r},\mathbf{t}) = \int G(\mathbf{r},t,\mathbf{r}',t')\sigma_j(\mathbf{r}',t')d^3r'dt'$$

The integral form depends on the boundary conditions. Two main cases are (a) for annually-periodic sources (so that the concentrations are periodic plus a globally uniform trend) and (b) prescribed initial conditions so that the inversions follow the full time-dependence. In each case, the fitting of concentrations can be expressed as a linear regression analysis, allowing uncertainties in the observations to be propagated through the calculations, thus giving uncertainties for the estimated sources. Generalising this to a Bayesian form allows the inversions to incorporate additional information to produce combined estimates based on multiple types of data. The Bayesian form has the additional advantage of reducing the effects of the ill-conditioning in the inversion. Comparing the posterior probability distributions to the prior distributions gives a measure of how much information is contibuted by the inversion. However, because these Bayesian inversions incorporate additional information through the prior distributions, care is needed to avoid 'double counting' when comparing the results to other estimates. For example, many inversions use the air-sea flux estimates from Takahashi et al. [15] as priors, albeit with large uncertainties.

Isotopes

In atmospheric CO_2 inversions, the isotope of most interest is ^{13}C. (^{14}C is important in ocean studies and ^{18}O is important in terrestrial studies.) The atmospheric budgets for total atmospheric carbon (M_A) and atmospheric ^{13}C (written as R_A M_A where R_A is the ^{13}C:C ratio) are

$$\frac{d}{dt}M_A = \sum \Phi_X = \sum[\Phi_x^+ - \Phi_x^-] \qquad \frac{d}{dt}(R_A M_A) = \sum(R_x^+\Phi_x^+ - R_x^-\Phi_x^-)$$

where Φ_x^+ and Φ_x^- are the one-way carbon fluxes from and to reservoir x.

In the ^{13}C budget, the contributions can be written:

$$R_x^+\Phi_x^+ - R_x^-\Phi_x^- = R_x^+(\Phi_x^+ - \Phi_x^-) + \Phi_x^-(R_x^+ - R_x^-) = R_x^+\Phi_x + \Phi_x^-(R_x^+ - R_x^-)$$

The first term on the right involves the net flux. The second term has become known as an isoflux and represents a contribution to ^{13}C change that occurs even in the absence of net carbon flux.

Having two budget equations allows us to solve for 2 unknowns. Most commonly, the technique has been applied to estimating the net flxues, Φ_O and Φ_B from the oceans and terrestrial biota. This 2-component budget formalism has been used in various ways, generally using terrestrial models to estimate

$$\Phi_B^-(R_B^+ - R_B^-):$$

Quay et al.[17] used a 'carbon-storage' form with changes in ocean ^{13}C determined by observations. Tans, Berry and Keeling [13] used observed air-sea isotopic disequilibria to estimate $\Phi_O^-(R_O^+ - R_O^-)$term.

Heimann and Maier-Reimer [8] produced an estimated budget based on combining the approaches of Quay et al,, Tans et al., and a third approach based on long-term isotopic relations. Francey et al. [7] and Keeling et al. [10] applied the Tans et al. form with time dependence. Enting [3] reviewed the statistical aspects of the Francey et al. analysis. Trudinger [16] has used a response function formalism, so that rather than a slowly-varying isoflux, the isoflux varies according to the actual isotopic fluxes.

Inversions with isotopes

There have been a number of studies that combine the principles of CO_2 inversions and carbon isotope budgeting

- Keeling et al. [9] performed a CO_2 synthesis and used ^{13}C distributions as a check, rather than as part of the estimation.
- Enting et al. [6]developed the Bayesian synthesis formalism for a steady-state inversion of CO_2 and ^{13}C data.
- 2-D mass balance inversions, simultaneously inverting CO_2 and ^{13}C, have been performed by Ciais et al. [1,2] and Morimoto et al. [11].
- Rayner et al. [18] performed a 3-D time-dependent inversion of CO_2 and ^{13}C.

Summary

Some illustrative results are:

- The main information about global budgets is in the global-scale concentrations.
- Enting [5] suggests that estimates from inversions can be presented as cumulative integrals over latitude, to reduce the degree of negative spatial autocorrelation.
- Inversions studies comparing synthesis inversion with 2 forms of mass balance inversion [19] suggest that estimates of interannual variability are relatively robust.

References

1. Ciais, P. et al. (1995) *Science,* **269**, 1098-1102.
2. Ciais, P. et al. (1995) *Journal of Geophysical Research*, **100D**, 5051-5070.
3. Enting, I.G., (1999) CSIRO Atmos. Res. Tech. Paper 40. *http://www.dar/csiro.au/publications/Enting_2000a.pdf*
4. Enting, I.G. (2000) in *Inverse Methods in Global Biogeochemical Cycles*. Ed. P.Kasibhatla et al. Geophysical Monography 114. (AGU. Washington).
5. Enting, I.G. (2000) *Inverse Problems in Atmospheric Constituent Transport.* (CUP) (in press).
6. Enting, I.G., Trudinger, C.M. and Francey, R.J. (1995) *Tellus*, **47B,** 35-52.
7. Francey, R.J. et al. (1995) *Nature*, **373,** 326-330.
8. Heimmann, M. and Maier-Reimer, E. (1996) *Global Biogeochem. Cycles,* **10**, 89-100.
9. Keeling, C.D., Piper, S. and Heimann, M. (1989) pp305 of *Aspects of Climate Variability in the Pacific and Western Americas.* Ed. D.H. Peterson. Geophysical Monography 55. (AGU: Washington).
10. Keeling, C.D. et al. (1995) *Nature,* **375,** 666-670.
11. Morimoto, A. et al. (2000) *Journal of Geophysical Research* **105D,** 24315-24326.
12. Tans, P.P., Fung, I.Y. and Takahashi, T. (1990) *Science*, **247**, 1431-1438.
13. Tans, P.P., Berry, J.A. and Keeling, R.F. (1993) *Global Biogeochem. Cycles,* **7,** 353-368.
14. Tans, P.P., Fung, I.Y. and Enting, I.G. (1995) p351 of *Biotic Feedbacks in the Global Climatic System.* Ed. Woodwell and Mackenzie. (OUP).
15. Takahashi, T. et al. (1999) p9-15 of *Proceedings of the 2nd Internatioal symposium in CO_2 in the Oceans.* Ed. Y. Nojiri. (Center for Global Environmental Research: Tsukuba).
16. Trudinger, C.M. (2000) Ph.D. Thesis, Monash University. Available at *http://www.dar.csiro.au/publications/Trudinger_2001a0.htm.*
17. Quay, P., Tillbrook, B. and Wong, C.S. (1992) *Science,* **256**, 74-79.
18. Rayner, P.J., Enting, I.G., Francey, R.J. and Langenfields, R. (1999) *Tellus,* **51B,** 213-232.
19. Rayner, P.J., Law, R.M. and Dargaville, R. (1999) *Geophys. Res. Lett.,* **26,** 493-496.
20. Sarmiento, J. and Sundquist, E. (1992) *Nature,* **356,** 589-593.

Notation

α_j Coefficient of basis functions $\sigma_j(\mathbf{r},t)$ in expansion of source/sink distribution.

$c(\mathbf{r},t)$ Carbon concentration at location **r** and time *t*.

$G(\mathbf{r},t,\ \mathbf{r}',t')$ Green's function defining the concentration at **r**,*t* due to a delta-function source at **r'**,*t'*.

M_A Amount of carbon in atmosphere.

r Position vector.

R_x The ^{13}C:C ratio of flux from process/reservoir *x*.

s(**r**,*t*) Net source minus sink at location **r** and time *t*.

t time

T [.] Transport operator.

Φ_x^+, Φ_x^- Carbon fluxes to and from the atmosphere from process or reservoir *x*.

$\Phi_x = \Phi_x^+ - \Phi_x^-$ Net carbon fluxes to the atmosphere from process or reservoir *x*.

$\sigma_j(\mathbf{r},t)$ The jth basis funtion used in expanding the source/sink distribution.